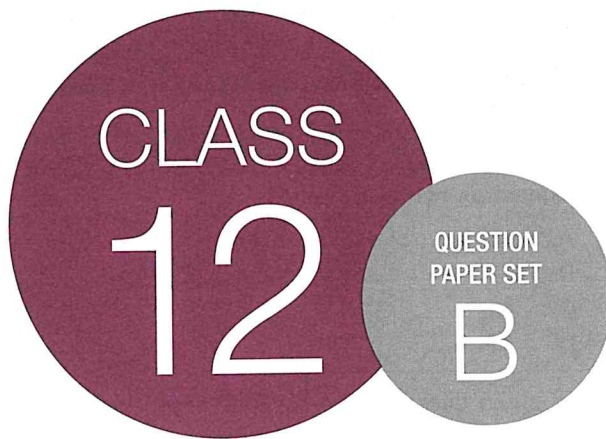


**SOF INTERNATIONAL
MATHEMATICS OLYMPIAD
2019-20**



DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Name:.....

Section:..... SOF Olympiad Roll No.:..... Contact No.:.....

Guidelines for the Candidate

- You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
- Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
- The Question Paper comprises four sections:
Logical Reasoning (15 Questions), **Mathematical Reasoning** (20 Questions), **Everyday Mathematics** (10 Questions) and **Achievers Section** (5 Questions)
 Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
- All questions are compulsory. There is no negative marking. Use of calculator is not permitted.
- There is only ONE correct answer. Choose only ONE option for an answer.
- To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.
 Q.16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is _____.
 A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg
 As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet. 16. ● (B) (C) (D)
- Rough work should be done in the blank space provided in the booklet.
- Return the OMR Sheet to the invigilator at the end of the exam.
- Please fill in your personal details in the space provided on this page before attempting the paper.



SCIENCE OLYMPIAD FOUNDATION
Inspiring Young Minds Through Knowledge Olympiads

LOGICAL REASONING

1. A cube is painted green on all faces and is then cut into 216 cubes of equal size. How many cubes are painted on two faces only?
- A. 16
B. 32
C. 48
D. 54

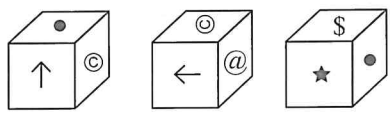
2. In given question two rows of numbers are given. The resultant number in each row is to be worked out separately based on the following rules. The operations on the numbers progress from left to right.

Rules:

- (i) If a two-digit odd number is followed by a prime number, then the first number is to be multiplied by the prime number.
- (ii) If an even number is followed by another even number, then the first number is to be divided by the second number.
- (iii) If an odd number is followed by a composite odd number, then the first number is to be added to the second number.
- (iv) If an even number which is a multiple of 5 is followed by another number which is also a multiple of 5, then the second number is to be subtracted from the first number.
- (v) If a number which is a perfect square is followed by another number which is also a perfect square, then the resultant number is the product of the square roots of the two numbers.

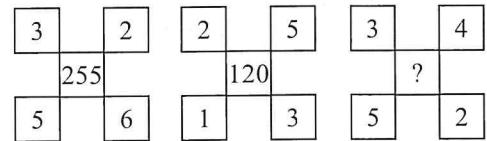
81	36	18
11	P	9

If P is the resultant of the first row, then what is the resultant of the second row?

- A. 44
B. 42
C. 43
D. 45
-
3. Three different positions of a dice are shown below. Find the symbol which is at the top, if ↑ is at the bottom.
- 
- A. ●
B. ©
C. \$
D. @
-
4. If in a certain code language, 'CHILDREN' is written as 'MKKGJBPC', then how will 'SHOWROOM' be written in the same language?

- A. XQKWQMLI
B. XQKWILMQ
C. WKQXILMQ
D. XJRWILMQ





5. Find the missing number, if same rule is followed in all the three figures.



- A. 196
B. 206
C. 195
D. 205

6. In which of the following options, the given figure is exactly embedded as one of its parts?



- A. 
- B. 
- C. 
- D. 

7. In a seminar, five professors U, V, W, X and Y participated. Study the given information and answer the question that follows.

- (i) V and W teaches Mathematics, although when X joined them they all changed to Physics, the only common subject among the three of them.
- (ii) The only common subject among U, V and Y was Chemistry.
- (iii) The only common subject between W and Y was English.
- (iv) Three professors cannot teach Economics.
- (v) The most common subject was Physics.
- (vi) One of the professors can teach all the five subjects, another one teach four, one teach three, one teach two and one teach only one subject.

Y definitely teaches which of the following subjects?

- I. Chemistry
II. English
III. Economics
- A. Only I
B. Both I and II
C. Both I and III
D. Both II and III

8. Select the correct water image of the given combination of letters, numbers and symbols.

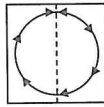
DL2CA#©43XY

- A. YX£†©#AÇÇJÇ
 B. DƆƆCV#©†3XΛ
 C. ΛX£†©#DL2CA#
 D. ΛX£†©#DL2CA#

9. Yash starts moving towards South and walks for 12 m. Then he takes the left turn and walks for 8 m. Again he takes a right turn and walks for 24 m. Now he takes a turn to North-East direction and walks 25 m. From that point he takes a turn towards North and walks 20 m. How far is he now from his starting point?

- A. 12 m
 B. 15 m
 C. 17 m
 D. 25 m

10. A square transparent sheet with a pattern and a dotted line is given. Select the figure from the options as to how the pattern would appear when the transparent sheet is folded along the dotted line.



- A. B.
 C. D.

11. The given question consists of three statements followed by three conclusions numbered I, II and III. Read all the conclusions and find which of the given conclusions logically follows from the given statements, if all statements are assumed to be true.

Statements :

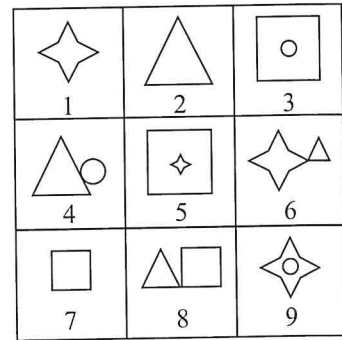
- (i) All plates are bowls
 (ii) Some plates are spoon
 (iii) No spoon is a jar

Conclusions :

- (I) Some bowls are plates
 (II) Some jars are spoon
 (III) Some bowls are spoons

- A. Only I is true
 B. Both II and III are true
 C. Both I and III are true
 D. None of these

12. Group the given figures into three classes on the basis of their identical properties using each figure only once.



- A. 1, 2, 6; 3, 4, 9; 5, 7, 8
 B. 1, 5, 6; 2, 4, 8; 3, 7, 9
 C. 1, 2, 9; 4, 6, 8; 3, 5, 7
 D. 1, 2, 7; 3, 5, 9; 4, 6, 8

13. Read the following information carefully and answer the question that follows.

- I. $P + Q$ means P is the father of Q.
 II. $P - Q$ means P is the wife of Q.
 III. $P \times Q$ means P is the brother of Q.
 IV. $P \div Q$ means P is the daughter of Q.

If $A \div C + D + B$, then which of the following statements is true?

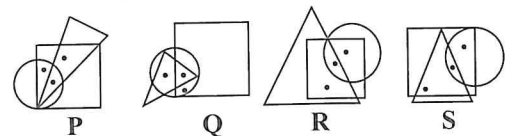
- A. A is the daughter of B
 B. B is the aunt of A
 C. A is the aunt of B
 D. A is the mother of B

14. How many 5's are there in the following sequence such that the product of the two immediately preceding digits is greater than the product of the two immediately following digits?

6 7 5 5 9 4 5 7 6 4 5 9 8 7 5 6 7 5 6 4 3 2 5 6 7 8
 5 4 2

- A. Two
 B. Three
 C. Four
 D. More than four

15. Which of the following two figures satisfies the same conditions of placement of dots?



- A. P and Q
 B. Q and R
 C. P and S
 D. Q and S

16. Find the number of ways of arranging the letters of the word KRISHNA such that all the vowels come together.

- A. 1280
- B. 1440
- C. 1320
- D. 1560

17. If $f(x) = \cos(\log x)$, then

$$f(x)f(y) - \frac{1}{2} \left\{ f\left(\frac{x}{y}\right) + f(xy) \right\} =$$

- A. 1
- B. 0
- C. -1
- D. None of these

18. If $|z - i \operatorname{Re}(z)| = |z - \operatorname{Im}(z)|$, (where $i = \sqrt{-1}$), then z lies on

- A. $\operatorname{Re}(z) = 2$
- B. $\operatorname{Im}(z) = 2$
- C. $\operatorname{Re}(z) + \operatorname{Im}(z) = 2$
- D. None of these

19. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis and lying in the first quadrant, is

- A. 9
- B. 36
- C. 18
- D. $27/4$

20. $\begin{vmatrix} a^2 + x^2 & ab & ca \\ ab & b^2 + x^2 & bc \\ ca & bc & c^2 + x^2 \end{vmatrix}$ is divisor of

- A. a^2
- B. b^2
- C. c^2
- D. x^2

21. Let R be a relation on the set of integers given by $a R b \Rightarrow a = 2^k b$ for some integer k . Then, R is

- A. An equivalence relation
- B. Reflexive only
- C. Reflexive and transitive but not symmetric
- D. Reflexive and symmetric but not transitive.

22. If $\log_2(5 \cdot 2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in A.P., then x equals

- A. $\log_2 5$
- B. $1 - \log_2 5$

- C. $\log_5 2$
- D. None of these

23. The degree and order of the differential equation of all parabolas, whose axis is x -axis are respectively

- A. 1 and 2
- B. 2 and 1
- C. 3 and 2
- D. 2 and 3

24. Calculate the possible values of x , if standard deviation of the numbers 2, 3, $2x$ and 11 is 3.5.

- A. -3, $-7/3$
- B. 3, $-7/3$
- C. -3, $7/3$
- D. 3, $7/3$

25. The probability that a teacher will give an unannounced test during any class meeting is $1/5$. If a student is absent twice, then the probability that he will miss at least one test, is

- A. $7/25$
- B. $9/25$
- C. $16/25$
- D. $24/25$

26. The equation of the ellipse whose distance between the foci is equal to 8 and distance between the directrices is 18, is

- A. $5x^2 - 9y^2 = 180$
- B. $9x^2 - 5y^2 = 180$
- C. $x^2 + 9y^2 = 180$
- D. $5x^2 + 9y^2 = 180$

27. If \vec{a} and \vec{b} are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and

$$\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}, \text{ then the value of } (2\vec{a} + \vec{b}) \cdot [\vec{a} \times (\vec{a} - 2\vec{b})]$$

is

- A. 2
- B. 3
- C. 4
- D. 0

28. Read the given statements carefully and select the correct option.

Let A be a 3×3 matrix with $|A| \neq 0$ such that $A^2 - 5A + 7I = O$.

Statement-I : $A^{-1} = \frac{1}{7}(5I - A)$

Statement-II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A + 4I)$.

- A. Both Statement-I and Statement-II are true.
 B. Statement-I is true but Statement-II is false.
 C. Statement-I is false but Statement-II is true.
 D. Both Statement-I and Statement-II are false.

29. If $\int x \log\left(1 + \frac{1}{x}\right) dx = f(x) \log(x+1) + g(x) \log_e x + Lx + C$, then

- A. $f(x) = \frac{x^2}{2}$
 B. $g(x) = \log_e x$
 C. $L = 1$
 D. $L = \frac{1}{2}$

30. The minimum value of $z = x + y$ subject to $2x + y \geq 5$; $2x + y \leq 10$; $3x \geq y$; $x \leq 2y$; $x, y \geq 0$ is

- A. 3
 B. 4
 C. 6
 D. 8

31. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$, then which one of the following is true?

- A. f is differentiable at $x = 0$ and at $x = 1$
 B. f is differentiable at $x = 0$ but not at $x = 1$
 C. f is differentiable at $x = 1$ but not at $x = 0$
 D. f is neither differentiable at $x = 0$ nor at $x = 1$

32. Which of the following statement is always correct?

- (i) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
 (ii) $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a tautology.

- A. Only (i)
 B. Only (ii)
 C. Both (i) and (ii)
 D. Neither (i) nor (ii)

33. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then the value of $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2}$ is

- A. $2xyz$
 B. $x^2 + y^2 + z^2$
 C. $xy + yz + zx$
 D. None of these

34. The acute angle between the line

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$$

$$\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 5$$
 is

- A. $\sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$ B. $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$
 C. $\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$ D. None of these

35. The value of c in Rolle's theorem when

$$f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [1/2, 3]$$
 is

- A. 2 B. $-\frac{1}{3}$
 C. -2 D. $\frac{2}{3}$

EVERYDAY MATHEMATICS

36. The mean age of a combined group of men and women is 25 years. If the mean age of the group of men is 26 and that of the group of women is 21, then the percentage of men and women respectively in the group is

- A. 46%, 60%
 B. 80%, 20%
 C. 20%, 80%
 D. 60%, 40%

37. Water flows through a cylindrical pipe of diameter 5 mm at the rate of 10 m per minute and falls into a conical vessel having 40 cm as the diameter of its base and 24 cm as its height. How long will it take to fill the vessel?

- A. 48 mins 15 secs
 B. 51 mins 12 secs
 C. 52 mins 1 sec
 D. 55 mins

38. A car travels 25 km an hour faster than a bus for a journey of 500 km. The bus takes 10 h more than the car. If speed of car is p and speed of bus is q , then

- A. $p = q^2$
 B. $p = 2q$
 C. $p = 3q$
 D. $p^2 = q$

39. If the goods be purchased for ₹ 672 and $\left(\frac{1}{4}\right)^{\text{th}}$ be sold at 20% loss, then at what gain% should sell the remaining part so as to make an overall profit of 15% on the total transaction?

- A. $\frac{80}{3}\%$ B. $\frac{70}{3}\%$
 C. 17.5% D. $\frac{50}{3}\%$

40. 9 children can complete a piece of work in 360 days. 18 men can complete the same piece of work in 72 days and 12 women can complete it in 162 days. In how many days can 4 men, 12 women and 10 children together complete the same piece of work?
- A. 68
B. 81
C. 96
D. 124
-
41. From the top of a cliff 90 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. What is the height of the tower?
- A. 30 m
B. 45 m
C. 60 m
D. 75 m
-
42. Four milkmen rented a pasture. A grazed 18 cows for 4 months, B grazed 25 cows for 2 months, C grazed 28 cows for 5 months and D grazed 21 cows for 3 months. If A's share of rent is ₹ 360, the total rent of the field is
- A. ₹ 1500
B. ₹ 1600
C. ₹ 1625
D. ₹ 1650
-
43. A started a business by investing ₹ 2000. After some time B joins the business by investing ₹ 1000. At the end of 12 months, the profit divided in the ratio 3 : 1. After how many months did B join the business?
- A. 6 months
B. 7 months
C. 4 months
D. 9 months
-
44. In a survey of 100 students, the number of students studying the various languages were found to be : English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. How many students were studying English and Hindi?
- A. 2
B. 5
C. 4
D. 3
-
45. Ratio of the earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8 : 7. What are A's earnings?
- A. ₹ 21,000
B. ₹ 26,000
C. ₹ 28,000
D. Data inadequate

ACHIEVERS SECTION

46. Match the integrals in Column-I with their values in Column-II and select the correct option.

Column-I	Column-II
(i) $\int_{-1}^1 \frac{dx}{1+x^2}$	(p) $\frac{1}{2} \log\left(\frac{2}{3}\right)$
(ii) $\int_0^1 \frac{dx}{\sqrt{1+x^2}}$	(q) $\log(1+\sqrt{2})$
(iii) $\int_2^3 \frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(iv) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

A. (i)-(s); (ii)-(q); (iii)-(p); (iv)-(r)
 B. (i)-(p); (ii)-(s); (iii)-(r); (iv)-(q)
 C. (i)-(s); (ii)-(p); (iii)-(q); (iv)-(r)
 D. (i)-(s); (ii)-(r); (iii)-(p); (iv)-(q)

47. Select the incorrect option.

A. Maximum value of $x(1-x)^2$, when $0 \leq x \leq 2$, is $\frac{4}{27}$.

- B. The minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$ is -128 .
 C. If the function $f(x) = x^4 - 62x^2 + ax + 9$ is maximum at $x = 1$, then the value of a is 120.
 D. None of these

48. Answer the following questions.

- (i) A person is known to speak the truth 4 times out of 5. He throws a die and reports that it is six. Find the probability that it is actually a six.
 (ii) A fair die is rolled. Consider the events $A = \{1, 3, 5\}$, $B = \{2, 3\}$ and $C = \{2, 3, 4, 5\}$. Then the conditional probability $P((A \cup B) / C)$ is

	(i)	(ii)
A.	$\frac{1}{3}$	$\frac{3}{4}$
B.	$\frac{4}{9}$	$\frac{1}{4}$
C.	$\frac{4}{9}$	$\frac{3}{4}$
D.	$\frac{5}{9}$	$\frac{1}{4}$

49. Find the value of p, q respectively, if

$$(a) \quad f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$$

is continuous in $[-1, 1]$.

$$(b) \quad f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16 + \sqrt{x}}) - 4}, & x > 0 \end{cases}$$

is continuous at $x = 0$.

- A. $8, -1/2$ B. $1/2, 0$
 C. $-1/2, 8$ D. $0, 1/2$

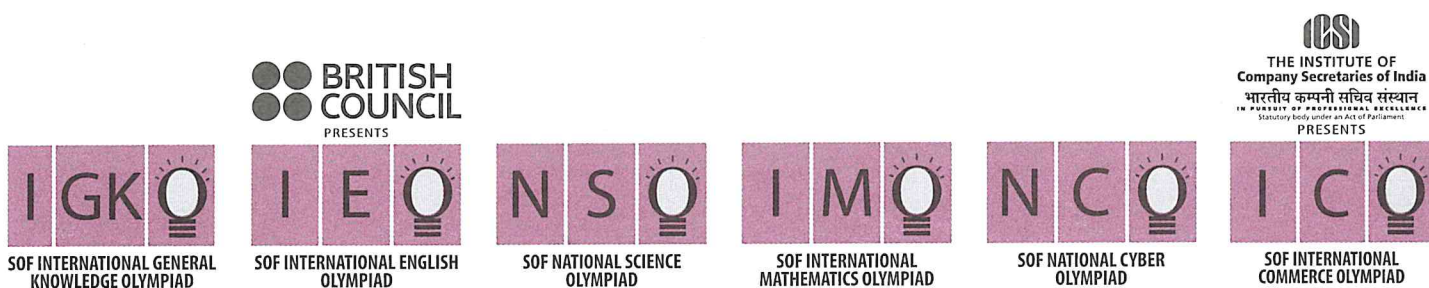
50. Read the given statements carefully and select the correct option.

Statement-I : The lines $\frac{x-1}{1} = \frac{y}{-1} = \frac{z+1}{1}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{3}$ are coplanar and the equation of the plane containing them is $5x + 2y - 3z - 8 = 0$.

Statement-II : The line $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z}{3}$ is perpendicular to the plane $3x + 6y + 9z - 8 = 0$ and parallel to the plane $x + y - z = 0$.

- A. Both Statement-I and Statement-II are true.
 B. Statement-I is true but Statement-II is false.
 C. Statement-I is false but Statement-II is true.
 D. Both Statement-I and Statement-II are false.

 SPACE FOR ROUGH WORK



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